Growth and decay of a convective boundary layer over a surface with a constant temperature

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BLLAST Workshop, Wageningen, 9 February 2016
A canonical case of atmospheric turbulence

- A very simple configuration
  - Linear stratification
  - A constant surface temperature
  - Buoyancy as thermodynamic variable
    \[ b = \frac{g}{\theta_v} (\theta_v - \theta_{v0}) \]
  - In total four parameters
- A detailed description of this reference case is lacking in literature
- Why do we want to study this?
  - Decay of turbulence: the afternoon transition
How does the system evolve in time?

- Surface flux reduces in time towards zero
- Height increases in time towards maximum height $L$
- Vertically integrated kinetic energy reaches peak and then decays

Magnitude of the temperature gradient vector
Derivation of a non-dimensional system and its two parameters

- **Outer length scale**
  \[ L \equiv \frac{b_0}{N^2} \]
  
  *largest length scale*

- **Buoyancy flux scale**
  \[ B \equiv b_0^4 \kappa^{\frac{1}{3}} \]
  
  *viscous scaling*

- **Velocity scale**
  \[ U \equiv (BL)^{\frac{1}{3}} = \frac{b_0^9 \kappa^{\frac{1}{9}}}{N^2 \kappa^{\frac{1}{3}}} \]
  
  *convective scale*

- **Time scale**
  \[ T \equiv \frac{b_0 L}{B} = \frac{b_0^2}{N^2 \kappa^{\frac{1}{3}}} \]
  
  *time to warm reservoir (volume / flux)*

\[
\begin{align*}
Pr & \equiv \frac{\nu}{\kappa}, \quad Re \equiv \frac{UL}{\nu} \\
\end{align*}
\]
Direct numerical simulations of four Reynolds numbers

- Direct numerical simulation
- Prandtl number of unity
- Four different Reynolds numbers
- Horizontal dimensions domain 2 x 2 m
- Domain height dependent on $L$

### Table 1: Overview of the numerical simulations

<table>
<thead>
<tr>
<th>Name</th>
<th>$N_x \times N_y \times N_z$</th>
<th>$b_0$</th>
<th>$b_0/N^2$</th>
<th>$\nu, \kappa$</th>
<th>$Re$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ReS</td>
<td>1024 $\times$ 1024 $\times$ 384</td>
<td>0.5</td>
<td>0.1667</td>
<td>$1 \times 10^{-5}$</td>
<td>285</td>
</tr>
<tr>
<td>ReM</td>
<td>1024 $\times$ 1024 $\times$ 768</td>
<td>1.0</td>
<td>0.3333</td>
<td>$1 \times 10^{-5}$</td>
<td>718</td>
</tr>
<tr>
<td>ReL</td>
<td>1536 $\times$ 1536 $\times$ 768</td>
<td>1.6</td>
<td>0.5333</td>
<td>$5 \times 10^{-6}$</td>
<td>2133</td>
</tr>
<tr>
<td>ReXL</td>
<td>2048 $\times$ 2048 $\times$ 1024</td>
<td>2.0</td>
<td>0.6667</td>
<td>$5 \times 10^{-6}$</td>
<td>2873</td>
</tr>
</tbody>
</table>
Time evolution of three relevant variables in all simulations

- Surface buoyancy flux decays towards zero
- Boundary layer depth evolves towards maximum given by $L$
- Integrated kinetic energy peaks early and then slowly decays
Non-dimensional results

- With scaling parameters $L$, $T$ and $B$ results can be plotted non-dimensional
- Non-dimensionalization leads to collapsing graphs
- Results of $ReL$ and $ReXL$ converge: extrapolation to atmosphere possible

$\text{surface buoyancy flux}$  $\text{boundary layer depth}$  $\text{integrated kinetic energy}$
Derivation of a mathematical model

- Our chosen boundary layer height is an integral length scale

\[ h^2_* \equiv \frac{2}{N^2} \int_0^\infty \langle b \rangle - N^2 z \, dz \]

- We can derive an exact expression for the rate of change of this length scale based on the simulation

\[ \frac{dh^2_*}{dt} = \frac{2}{N^2} (B_s + \kappa N^2) \]
Non-dimensional form of the mathematical model

- Differential equation is the same as that for a bulk model without entrainment (encroachment model)
  \[
  \frac{dh_*}{dt} = \frac{B_s + \kappa N^2}{h_* N^2}
  \]

- Equation can be rewritten in non-dimensional variables with the help of the defined length, time and buoyancy scales \( L, T \) and \( B \)
  \[
  \frac{\hat{dh}_*}{dt} = \frac{\hat{B}_s + Re^{-\frac{3}{4}}}{\hat{h}_*}
  \]

- For high Reynolds numbers, second term in numerator can be neglected
  \[
  \frac{\hat{dh}_*}{dt} = \frac{\hat{B}_s}{\hat{h}_*}
  \]
A model for the surface buoyancy flux is needed

- We propose a model for the surface buoyancy flux
  - The mixed-layer value is approximated by $\min(b)$

\[
B_s \equiv c_0 \kappa^{\frac{1}{3}} (b_0 - \min(b))^{\frac{4}{3}}
= c_0 \kappa^{\frac{1}{3}} (b_0 - c_1 h_\ast N^2)^{\frac{4}{3}}
\]

- The values for the constants can be derived from the simulations
Final form of the mathematical model for our system

• Substitution of the analytical model for the surface buoyancy flux gives

\[
\frac{dh_*}{dt} = \frac{c_0 \kappa^{\frac{1}{3}} (b_0 - c_1 h_* N^2)^{\frac{4}{3}}}{h_* N^2}
\]

• In terms of non-dimensional variables this is

\[
\frac{\hat{d}h_*}{\hat{d}t} = \frac{c_0 \left(1 - c_1 \hat{h}_*\right)^{\frac{4}{3}}}{\hat{h}_*}
\]

• This can be solved analytically and all the scaling variables can be derived
Validation of the derived mathematical model

- The model matches very well with the data
- Nearly perfect match for the two highest Reynolds number cases
- Model predicts decay rate of kinetic energy in a system that slowly dies out

![Graphs a), b), c) showing the relationship between surface buoyancy flux, boundary layer depth, and integrated kinetic energy over time.](image)
Back to typical atmospheric dimensions

- Typical atmospheric conditions
  - Excess surface temperature 6 K, lapse rate 6 K km\(^{-1}\), thus \(L = 1000\) m
  - Three exchange rates: smooth, moderately rough and very rough surface

- Decay does not follow a power law, but has increasingly negative slope
Conclusions

• The convective boundary layer over surface with a fixed surface temperature
  • Complex transient system with a peak in integrated kinetic energy

• Direct numeral simulations of system can be extrapolated to atmosphere
  • Reynolds number similarity presented

• A model derived for high Reynolds number flows
  • Mathematical model is able to predict bulk characteristics of the system
  • Model can be used to predict the afternoon transition of the CBL

• The decay of kinetic energy in the boundary layer does not follow power law